Guide to the Expression of Uncertainty in Measurement (GUM) and its supplemental guides

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Outline

JCGM

Current GUM

GUM revision

Two phases of uncertainty evaluation

Supplemental guides
Joint Committee for Guides in Metrology (JCGM)

JCGM comprises

- Bureau International des Pois et Mesures (BIPM)
- International Electrotechnical Commission (IEC)
- International Federation of Clinical Chemistry (IFCC)
- International Organisation for Standardisation (ISO)
- International Union of Pure and Applied Chemistry (IUPAC)
- International Union of Pure and Applied Physics (IUPAP)
- International Organisation of Legal Metrology (OIML)
- International Laboratory Accreditation Cooperation (ILAC)

Body responsible for GUM revision

JCGM/WG1, Measurement Uncertainty
Guide to the Expression of Uncertainty in Measurement

GUM published by ISO in mid-1990s

Widely used and respected

Recognised by many as the master document on uncertainty

Embraces many aspects of uncertainty evaluation

But ... some deficiencies and limitations identified

Process of GUM revision under way
Why revise the GUM?

Improve ease of use

Give added value

Address limitations
Politics

Continue to promote current GUM

Enormous investment by organisations

Make no change to GUM *per se*

Added value through supplemental guides
The two phases of uncertainty evaluation

1. Formulation
   - Develop (mathematical or other) model
   - Assign input probability distributions

2. Calculation
   - Derive output probability distribution
   - Estimate measurand and standard uncertainty
   - Provide coverage interval for measurand
     - E.g., at 95% level of confidence
Current GUM

Commonest implementation of GUM: LPU + CLT

[GUM Clause 8]

Best estimate $y$ of measurand $Y$

Model evaluated at best estimates $x_i$ of input quantities $X_i$

LPU: Law of Propagation of Uncertainty

Combination of standard uncertainties $u(x)$ (and covariances) of input quantities $→$ standard uncertainty $u(y)$ of $y$

CLT: Central Limit Theorem

Assign a Gaussian (or “$t$”) distribution to the measurand
Current GUM

Implementations

Higher-order LPU possible
  Difficult to implement

GUM provides formula for mutually independent case
  Assumes Gaussian

No GUM formula for mutually dependent case

Software packages often apply LPU + CLT

Checks for applicability?

Fitness for purpose
Supplemental guides

1. Propagation of distributions
2. More than one measurand
3. Conformity assessment
4. Modelling
5. Calibration curves
1. Propagation of distributions

Required to ensure valid solution when

- Contributory uncertainties arbitrarily large
- Contributory uncertainties not comparable in size
- Very few contributory uncertainties
- Models highly nonlinear or complicated
- Distributions asymmetric
1. Propagation of distributions

Recommended implementation procedure:
Monte Carlo simulation
Very simple
Avoids partial derivatives!
1. Very simple non-linear model: \( Y = X_1^2 \)

\[ x_1 = 1, \quad u(x_1) = 1 \]

Normal input does not imply normal output when model is non-linear

Applicability of Central Limit Theorem!
1. Validation of uncertainty calculations

Use MCS to validate LPU + CLT

Favourable comparison
   LPU + CLT satisfactory for application

Unfavourable comparison
   Either detailed investigation required
   Or use MCS instead for uncertainty calculation
2. More than one measurand

\[ Y_1 = f_1(X_1, X_2, X_3) \]
\[ Y_2 = f_2(X_1, X_2, X_3) \]

Input quantities \( X_1, X_2, \ldots \)
Values, standard uncertainties and covariances

Output quantities \( Y_1, Y_2, \ldots \)
Values, standard uncertainties and covariances
\( Y_1, Y_2, \ldots \) almost invariably correlated
2. More than one measurand

Generalisations of the GUM formulae

Matrix-vector expressions

Compact formulae

Suitable for software implementation

Provision of best estimates of $Y_1$, $Y_2$, ..., and standard uncertainties and covariances

Coverage regions under consideration!

$E.g., \ V_y = J_x V_x J_x^T$
3. Uncertainty in conformance testing

Industrial inspection: tolerance zones
Regulations: Limit values
Process and inherent variability
Decisions based on measurement
3. Uncertainty in conformance testing

Accounting for measurement uncertainties
Acceptance and rejection of components
Repercussions of surpassing regulatory limits
Decision rules
Consumer’s and producer’s risks
4. Modelling

The formulation phase of uncertainty evaluation
Physical, empirical and hybrid models
Functional and probabilistic models

Assignment of probability distributions to input quantities

Classification of models
Explicit or implicit
Real or complex-valued
1 or more than 1 measurand

\[ 2 \times 2 \times 2 = 8 \text{ categories} \]
4. Modelling

Multistage models
Managing multistage models
Example: calibration curves

Measurement data → Calibration curve parameters → Calibration curve values → Area under calibration curve
5. Least squares adjustment

Least-squares modelling

Widespread in metrology

Associated uncertainty evaluation

Not well understood

Calibration curves a very important application

Response vs. stimulus

Generation of curve from standards (response values)

Use of curve to obtain stimulus values
5. Calibration curves

Given stimuli (standards $x_i$ & $u(x_i)$)
Measure responses $y_i$ & obtain $u(y_i)$
Derive regression line (curve) $y = f(x)$
Form $x$ & $u(x)$ given $y$ & $u(y)$

Respect stated uncertainties and known mutual dependencies

JCGM and ISO/TC69 link
Conclusion

Respect investment in common use of GUM

Give added value to current GUM

Two phases of uncertainty evaluation

Supplemental guides

1. Propagation of distributions
2. More than one measurand
3. Uncertainty in conformance testing
4. Modelling
5. Least-squares adjustment
Optional slides
Uncertainty-related standards

- JCGM
  - GUM revision
  - BIPM Director’s Adv Group on Uncertainties
  - MRA

- ISO
  - Appln of Statl Methods
  - WG: Relation ISO5725 & GUM

- CEN
  - Unc of measurement

- EA
  - Unc experts’ group

- BSI
  - Precision of test methods
  - Measurement unc

Terms of ref

= convenorship
Other new work items in JCGM

Introductory document on uncertainty

Introduce the GUM and its supplemental guides

“Industrial” guide to uncertainty
Introductory document on uncertainty

Possible scope

An introduction to measurement uncertainty
An introduction to the GUM
An introduction to GUM supplemental guides
Introductory document on uncertainty

Emphasise intention of the GUM

Provide an estimate (the measurement result) of a quantity of interest (the measurand)

Provide a coverage interval about the estimate that can be expected to contain a specified large fraction of the possible values of the measurand
Means of achieving aim

Given

The effects on which the measurand depends

Probability distributions assigned to these effects

A rule/functional relationship/model for expressing the measurand in terms of these effects

Determine probability distribution for the measurand →

the measurement result

the associated standard uncertainty

a coverage interval for the measurand
Audience

Many bodies, including

Academia
Industry
Accreditation bodies
Approach

Describe scope of GUM and supplemental guides
Help to the reader to navigate through them

Three considerations
- Concepts
- Formulation
- Implementation

Particular GUM implementation
- LPU + CLT
Discussion